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Bending of Rectangular Plates with Finite Deflections

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ABSTRACT

A previously developed iterative procedure is applied to obtain numerical solutions of the von Kármán equations for rectangular plates subjected to a uniform normal pressure. On the simply supported boundary it is assumed that the normal membrane stress and the tangential membrane displacement vanish. Solutions are obtained for a wide range of values of the loading parameter and the aspect ratio. Boundary layers develop both as the loading parameter and the aspect ratio increase. The stresses and deflections are examined and compared with an "asymptotic" solution which can be valid only in the interiors of long plates. A comparison is made with a previously obtained approximate solution by the Ritz method for the square plate.

1. Introduction

A thin, elastic rectangular plate is deformed by a uniform lateral pressure p applied normal to one face. The stresses and deflections predicted by the classical linear bending theory are valid only for a limited range of low pressures. For larger values of p it is necessary to employ nonlinear theories which allow for finite deflections and account for the interaction of membrane and bending effects. We employ the von Kármán plate theory [1] which is obtained from a nonlinear elasticity theory that permits small strains and small but finite displacements.

We consider the von Kármán equations as a coupled system of four second order partial differential equations. The four dependent variables are proportional to the normal displacement of the midsurface, the stress function and the Laplacians of these quantities. On the simply supported boundary it is assumed that the normal membrane stress and the tangential membrane displacement vanish. The stress strain and strain displacement relations of the von Kármán theory then imply that the four dependent variables vanish on the boundary. The solutions of this boundary value problem for the von Kármán equations depend on two parameters, the aspect ratio ℓ and a loading parameter P which is proportional to p.

In this paper, numerical solutions are obtained by an "accelerated" iteration method. It is analogous to one previously employed in studies of the nonlinear axisymmetric bending and buckling of circular plates [2] and the buckling of compressed rectangular plates [3]. Each iterate is determined as the solution of Dirichlet's problem for Poisson's equation on a rectangle. They are numerically evaluated by approximating the Dirichlet problem by corresponding difference equations. The resulting system of linear algebraic equations, which is of quasi-tridiagonal form, is solved by a factoring method [4]. The development of boundary layers is studied by obtaining solutions for a wide range of parameters.

The present results are compared with approximate solutions of a similar boundary value problem for the square plate obtained

by a Ritz method [5]. Other boundary conditions were employed in previous investigations of the nonlinear bending of rectangular plates. Approximate solutions were obtained by perturbation methods [6], energy methods [7], infinite series e.g. [8], and finite difference methods [9-11]. In these investigations relatively low values of P are considered and boundary layer phenomena are not studied. The finite difference procedures [9-11], which in some aspects are related to our method, employ relatively few mesh points.

2. Formulation

The rectangular plate of thickness t occupies the region: $0 \le X \le a$, $0 \le Y \le b$ and $|Z| \le t/2$. It is deformed by a uniform pressure p applied normal to one face and directed in the positive Z direction. The displacements of the midsurface $Z = 0$ in the X, Y and Z directions are denoted by $U(X,Y)$, $V(X,Y)$ and $W(X,Y)$. The membrane stresses are called $\sigma_x(X,Y)$, $\sigma_y(X,Y)$ and $\sigma_{xy}(X,Y)$.

The following dimensionless variables are defined by:

$$x \equiv Xb^{-1} \ , \ y \equiv Yb^{-1} \ , \ 0 \le x \le \ell \equiv a/b \ , \ 0 \le y \le 1 \ ,$$

$$u(x,y) \equiv c^2bt^{-2}U(X,Y) \ , \ v(x,y) \equiv c^2bt^{-2}V(X,Y) \ ,$$

$$(2.1) \qquad w(x,y) \equiv ct^{-1}W(X,Y) \ , \ c^2 \equiv 12(1-\nu^2) \ , \ P \equiv \frac{c^3}{E}\left(\frac{b}{t}\right)^4 p \ ,$$

$$\Sigma_x(x,y) \equiv \frac{c^2}{E}\left(\frac{b}{t}\right)^2\sigma_x(X,Y) \ , \ \Sigma_y(x,y) \equiv \frac{c^2}{E}\left(\frac{b}{t}\right)^2\sigma_y(X,Y) \ ,$$

$$\Sigma_{xy}(x,y) \equiv \frac{c^2}{E}\left(\frac{b}{t}\right)^2\sigma_{xy}(X,Y) \ ,$$

where ν is Poisson's ratio and E is Young's modulus. The stress function, $f(x,y)$, is defined in terms of the dimensionless membrane stresses Σ_x, Σ_y and Σ_{xy} by

$$\Sigma_x = f,_{yy} \ , \ \Sigma_y = f,_{xx} \ , \ \Sigma_{xy} = -f,_{xy} \ . \tag{2.2}$$

A comma denotes partial differentiation with respect to the subscripted variables following the comma.

The dimensionless membrane stresses are given in terms of the dimensionless displacements u, v and w by [1]

$$\begin{aligned} \Sigma_x - \nu\Sigma_y &= u,_x + \tfrac{1}{2}w,_x^2 \ , \quad \Sigma_y - \nu\Sigma_x = v,_y + \tfrac{1}{2}w,_y^2 \ , \\ \Sigma_{xy} &= [2(1+\nu)]^{-1}[u,_y + v,_x + w,_x w,_y] \ . \end{aligned} \tag{2,3}$$

The moments M_x, M_y and M_{xy} and the corresponding dimensionless moments m_x, m_y and m_{xy} are given in terms of the dimensionless displacement, $w(x,y)$, by

$$\begin{aligned} m_x(x,y) &\equiv -\frac{c^3b^2}{Et^4} M_x(X,Y) = w,_{xx} + \nu w,_{yy} \ , \\ m_y(x,y) &\equiv -\frac{c^3b^2}{Et^4} M_y(X,Y) = w,_{yy} + \nu w,_{xx} \ , \\ m_{xy}(x,y) &\equiv \frac{c^3b^2}{(1-\nu)Et^4} M_{xy}(X,Y) = w,_{xy} \ . \end{aligned} \tag{2.4}$$

The functions $\Omega(x,y)$ and $\phi(x,y)$ are defined by

$$\Omega(x,y) \equiv \triangle\, w(x,y) \ , \ \phi(x,y) \equiv \triangle\, f(x,y) \tag{2.5}$$

where Δ is the Laplacian with respect to x and y. Then the differential equations of the von Kármán theory can be written as

$$\Delta\phi = K[w] \,, \qquad \Delta f = \phi \,,$$

(2.6a)

$$\Delta\Omega = P + H[w,f] \,, \qquad \Delta w = \Omega \,.$$

Here the functionals K and H are defined as

$$K[w] \equiv w,_{xy}^{2} - w,_{yy}w,_{xx} \,,$$

(2.6b)

$$H[w,f] \equiv f,_{yy}w,_{xx} + f,_{xx}w,_{yy} - 2f,_{xy}w,_{xy} \,.$$

On the boundary B of the rectangular region D, we require that

$$\left.\begin{aligned} &w = \Delta w = 0 \,, && \text{(2.7a)} \\ &f = \Delta f = 0 \,, && \text{(2.7b)} \end{aligned}\right\} \quad (x,y) \text{ on } B \,.$$

The conditions (2.7a) and (2.4) imply that the boundary of the plate is simply supported. The boundary conditions (2.7b) imply, using (2.2) and (2.3), that along each edge of the boundary the normal stress vanishes and the tangential displacement is a constant which we take as zero.

The complete formulation of the boundary value problem consists of the differential equations (2.6) and the boundary conditions (2.7). The solutions depend on the two parameters

P and ℓ. When a solution (w,f) is obtained the dimensionless stresses and moments are computed from (2.2) and (2.4). The displacements u and v are then determined from (2.3) using the condition that the tangential displacements vanish on the boundary.

3. Numerical Methods

Approximate solutions of the boundary value problem (2.6), (2.7) are obtained by an accelerated iteration procedure [3]. Thus, starting from an initial estimate $w^{(o)}(x,y)$ of the solution for fixed P and ℓ, a sequence of iterates, $[\phi^{(n)}(x,y), f^{(n)}(x,y), \Omega^{(n)}(x,y), w^{(n+1)}(x,y)]$, is defined by the recursions,

$$
(3.1)\qquad
\begin{aligned}
&\Delta\phi^{(n)} = K[w^{(n)}] \,, && \phi^{(n)} = 0 \text{ on } B \,,\\
&\Delta f^{(n)} = \phi^{(n)} \,, && f^{(n)} = 0 \text{ on } B \,,\\
&\Delta\Omega^{(n)} = P + H[w^{(n)}, f^{(n)}] \,, && \Omega^{(n)} = 0 \text{ on } B \,,\\
&\Delta\overline{w}^{(n+1)} = \Omega^{(n)} \,, && \overline{w}^{(n+1)} = 0 \text{ on } B \,,\\
&w^{(n+1)} = \theta\overline{w}^{(n+1)} + (1-\theta)w^{(n)} \,.
\end{aligned}
$$

The acceleration parameter θ is to be determined so that the iterations converge as rapidly as possible. Each iterate in (3.1) is thus determined as the solution of a linear boundary value problem of the form,

(3.2) $$\Delta g(x,y) = c(x,y)\ ,\ g = 0 \text{ on } B\ ,$$

where $c(x,y)$ is a known function on the rectangle D. The solution $g(x,y)$ of (3.2) is determined approximately using a difference method. To apply this method, D is covered by a uniform rectilinear net with spacing δ in the x and y directions such that the boundary lines of D coincide with net lines. The net points (x_i,y_j) are given by

(3.3a) $$x_i = i\delta\ ,\ y_j = j\delta\ ,\ i = 0,1,\ldots,M\ ,\ j = 0,1,\ldots,N\ ,$$

where

(3.3b) $$\delta \equiv \ell/M = 1/N\ .$$

At each net point (x_i,y_j) interior to B, it is assumed that the solution of (3.2), $g(x_i,y_j)$, is approximated by the net function $\{g_{ij}\}$ which satisfies the difference equations

(3.4a) $$\Delta_\delta g_{ij} = c_{ij}$$

where

(3.4b) $$\Delta_\delta g_{ij} \equiv \frac{1}{6\delta^2}\,[g_{i+1,j+1} + g_{i-1,j-1} + g_{i-1,j+1} + g_{i+1,j-1} + 4(g_{i,j+1} + g_{i-1,j} + g_{i,j-1} + g_{i+1,j}) - 20g_{i,j}]$$

is the nine point Laplace difference operator [12]. The net functions $\{c_{ij}\}$ in (3.4a) are obtained by replacing the second derivatives occurring in the right hand sides of (3.1) by

centered second difference approximations. The net function $\{g_{ij}\}$ vanishes at each net point which lies on B.

The coefficient matrix of the (M-1)×(N-1) algebraic equations (3.4) is tridiagonal with respect to matrices or of quasi-tridiagonal form. This system is solved by factoring the matrix into the product of two matrices, one lower and one upper triangular with respect to matrices and then successively inverting the triangular systems [4]. The N-1, (M-1)×(M-1) inverses that are thus required are obtained by Gauss elimination with pivotal condensation. For a fixed δ, these inverses are computed only once. Hence, only matrix multiplications of vectors and evaluations of the inhomogeneous terms in (3.1) are required to determine an iterate.

4. Computational Methods*

The N-1 inverses are stored in the internal or fast access memory, thus limiting the size of the net.† Hence for a square plate we are essentially limited to 625 interior net points ($\delta = 1/26$) and for a rectangular plate with $\ell = 2$ to 741 points ($\delta = 1/20$). However, a sequence of test calculations with successively finer nets indicates that they are adequate for the range of P that we consider, see e.g. Table III below.

As an initial iterate we use, for any P and a fixed value

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*All computations were performed on the IBM-7090 and 7094 computers at the A.E.C. Computing and Applied Mathematics Center of the Courant Institute of Mathematical Sciences.

†Finer nets can be employed if tape storage is used. This considerably increases the computing time.

of ℓ, the numerical solution for a neighboring value of P. The first initial iterate is obtained for P near zero. Small pressures deform the plate slightly from the plane. Therefore, we take

$$\left\{w_{ij}^{(o)}\right\} = 0 \ , \ i = 1,2,\ldots, M-1, \ j = 1,2,\ldots N-1 \ ,$$

and the iterations converge rapidly. The converged solution is then used as an initial iterate for slightly larger P. In this manner numerical solutions are obtained for an increasing sequence of loads.

As a numerical convergence criterion we require

$$(4.1) \qquad R_n \equiv \max_{\substack{1 \leq i \leq M-1 \\ 1 \leq j \leq N-1}} \left|\bar{w}_{ij}^{(n+1)} - w_{ij}^{(n)}\right| < \varepsilon$$

where $\varepsilon > 0$ is a prescribed "small" number. For the calculations, epsilons were in the range $1 \times 10^{-9} < \varepsilon < 6.5 \times 10^{-6}$ and we usually employed the smallest ε consistent with the accuracy of the computational arithmetic. It should be noted that $\lim_{n \to \infty} R_n = 0$ is only a necessary condition for convergence. When the iterations have converged for specified values of P and ℓ, the dimensionless stresses and moments are computed from difference equivalents of (2.2) and (2.4).

In general, the number of iterations necessary to satisfy (4.1) increases as P increases and depends on the value of θ used in (3.1). The optimal value of θ, θ_o, is defined as that

value of θ which, for a fixed P and ℓ, minimizes the number of iterations that are required to satisfy (4.1). Estimates of θ_o are obtained from a series of test calculations with coarse meshes. The estimated θ_o is found to decrease as P increases.

The number of iterations needed for convergence can be decreased if the acceleration parameter in (3.1) is allowed to vary with x and y. To illustrate this, numerical experiments were conducted for the square plate with P = 100,000. The converged solution for P = 95,000 was employed as the initial iterate and a sequence of calculations were made for the following choices of acceleration parameters: 1) θ = .033 for all net points; 2) θ = .066 at all points on a band around the edge one net line in from the boundary, θ = .033 elsewhere; 3) θ = .066 at all net points on a band around the edge consisting of the first two net lines in from the boundary, θ = .033 elsewhere; 4) θ = .066 at the four corner net points which are on a band one line in from the boundary, θ = .033 elsewhere; 5) θ = .066 at the same four corner net points and their two nearest interior net point neighbors, θ = .033 elsewhere. The number of iterations needed for convergence for each of the five test calculations is summarized in Table I.

TABLE I

Test Number	1	2	3	4	5
Number of Iterations	182	164	171	156	153

5. Presentation and Discussion of Results[*]

Numerical solutions were obtained for square plates and a sequence of loads in the range $0 \leq P \leq 100{,}000$ and for rectangular plates with $\ell = 2$ and loads in the range $0 \leq P \leq 6{,}000$. Solutions were also obtained for plates with $\ell = 4$, 5, 6, 8, 10 and $P = 1{,}000$ and 2,000. The final mesh widths that were employed for each ℓ are summarized in Table II.

TABLE II

ℓ	1	2	4	5	6	8	10
$1/\delta$	26	20	16	14	12	10	10

Only one numerical solution of the boundary value problem was determined for each P and ℓ considered. Each solution is symmetric with respect to the lines $x = \ell/2$ and $y = 1/2$.

To estimate the range of validity of the classical linear bending theory[**] [13], the variation with P for "low" loads of the normal deflection of the plate's center is shown in Fig. 1. The graphs indicate that the central deflections predicted by the linear theory [13] deviate from those of the nonlinear theory, for example, for the square plate, by more than 10% if $P \geq 500$ and the difference increases rapidly with P.

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[*] All calculations employed $\nu = .3$.

[**] This theory is obtained from (2.6a) and (2.7) by setting $H[w,f] = K[w] \equiv 0$ in (2.6).

Representative graphs of the variation with x at $y = 1/2$ of dimensionless stresses and displacements are presented in Fig's. 2 for the square plate and an increasing sequence of loads. For convenience†, the results are shown for one half of the plate. The normal displacement, Fig. 2a, attains its maximum at the center for all loads considered. The plate "flattens" slightly at the center for the larger loads and the flattening effect increases as P increases. The formation of boundary layers as P increases is more evident in the graphs shown in Fig. 2b. For low loads $m_x(x, 1/2)$ attains its maximum at the center. For larger values of P there is a local minimum at the center. The maximum, which then occurs between the center and the edge, moves towards the edge as P increases. A similar boundary layer development occurs for the membrane stress Σ_y, see Fig. 2c. The variations of the shear stress Σ_{xy} and the twisting moment m_{xy} near the edge, i.e. at $y = 1/26$, are presented in Fig's. 3. The position of the maximum shear stress moves towards the corner as P increases which indicates the formation of boundary layers.

Boundary layers are found to form more rapidly and at lower loads for the rectangular plate with $\ell = 2$ than for the square plate.

To study the variations of the solutions with ℓ, new variables are defined by

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† Symmetry conditions imply that it is necessary to consider only one eighth of the square plate.

$$(5.1) \qquad x' \equiv x - \ell/2 \,, \qquad w'(x', y) = w(x' + \ell/2, y) \,.$$

In Fig's. 4, $w'(0,y)$ and $w'(x', 1/2)$ are shown for $P = 2000$ and an increasing sequence of aspect ratios. The "asymptotic" solution, indicated by the dashed curves in Fig's. 4, is obtained by first substituting (5.1) into the boundary value problem (2.6) and (2.7) and then letting $\ell \longrightarrow \infty$. The limit plate is then contained in the infinite strip, $|x'| < \infty$, $0 \leq y \leq 1$. The asymptotic solution, $w_A(y)$, which is independent of x', is defined as the solution of the limiting boundary value problem and is found to be,

$$(5.2) \qquad w_A(y) = \frac{Py}{24}(y^3 - 2y^2 + 1) \,, \quad f_A(y) \equiv 0 \,.$$

It does not satisfy all the boundary conditions (2.7) on $x' = \pm \ell/2$. Hence w_A may be an approximation to the solution of (2.6), (2.7) for "long" plates only away from the ends. More accurate approximations could be obtained by applying appropriate boundary layer methods.

A comparison is given in Table III for $P = 92{,}656.1$ of the center deflections $w(1/2, 1/2)$ obtained from a four term Ritz calculation for the square plate [5] and those obtained by the present method for successively finer meshes.

TABLE III

Net Size	$\delta = 1/26$	$\delta = 1/12$	$\delta = 1/6$	4 Term Ritz[5]
$w(1/2,1/2)$	22.6699	21.3592	18.9430	18.7195

Similar comparisons can be made for other loads. The maximum deviation between the Ritz results and the present ones for the finer mesh $\delta = 1/26$ is not at the center.

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Captions for Figures

Fig. 1. The variations with P of the normal deflections at the center of the square, $\ell = 1$, and rectangular plate, $\ell = 2$.

Fig. 2a. The variation of $w(x,1/2)$ with x for the square plate and an increasing sequence of values of P.

Fig. 2b. The variation of $m_x(x,1/2)$ with x for the square plate and an increasing sequence of values of P.

Fig. 2c. The variation of $\Sigma_y(x,1/2)$ with x for the square plate and an increasing sequences of values of P.

Fig. 3a. The variation of $\Sigma_{xy}(x,1/26)$ with x for the square plate and an increasing sequences of values of P.

Fig. 3b. The variation of $m_{xy}(x,1/26)$ with x for the square plate and an increasing sequence of values of P.

Fig. 4a. The variation of $w'(0,y)$ with y for an increasing sequence of values of ℓ with P = 2000 and a comparison with the asymptotic solution $w_A(y)$, which is indicated by the dashed curve, see (5.1) and (5.2).

Fig. 4b. The variation of $w'(x',1/2)$ with x' for an increasing sequence of values of ℓ with P = 2000 and a comparison with the asymptotic solution which is indicated by the dashed curve.

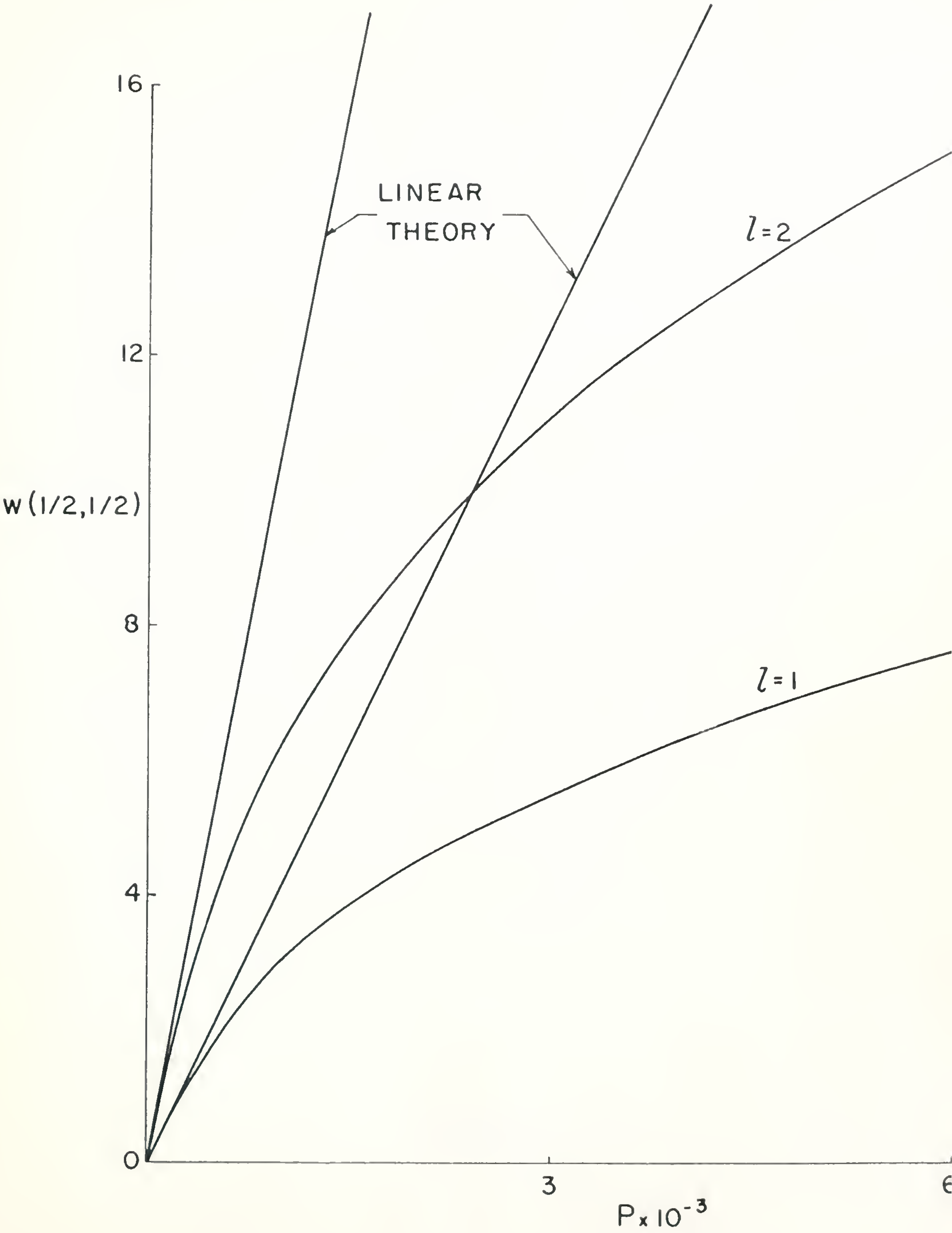

Figure 1

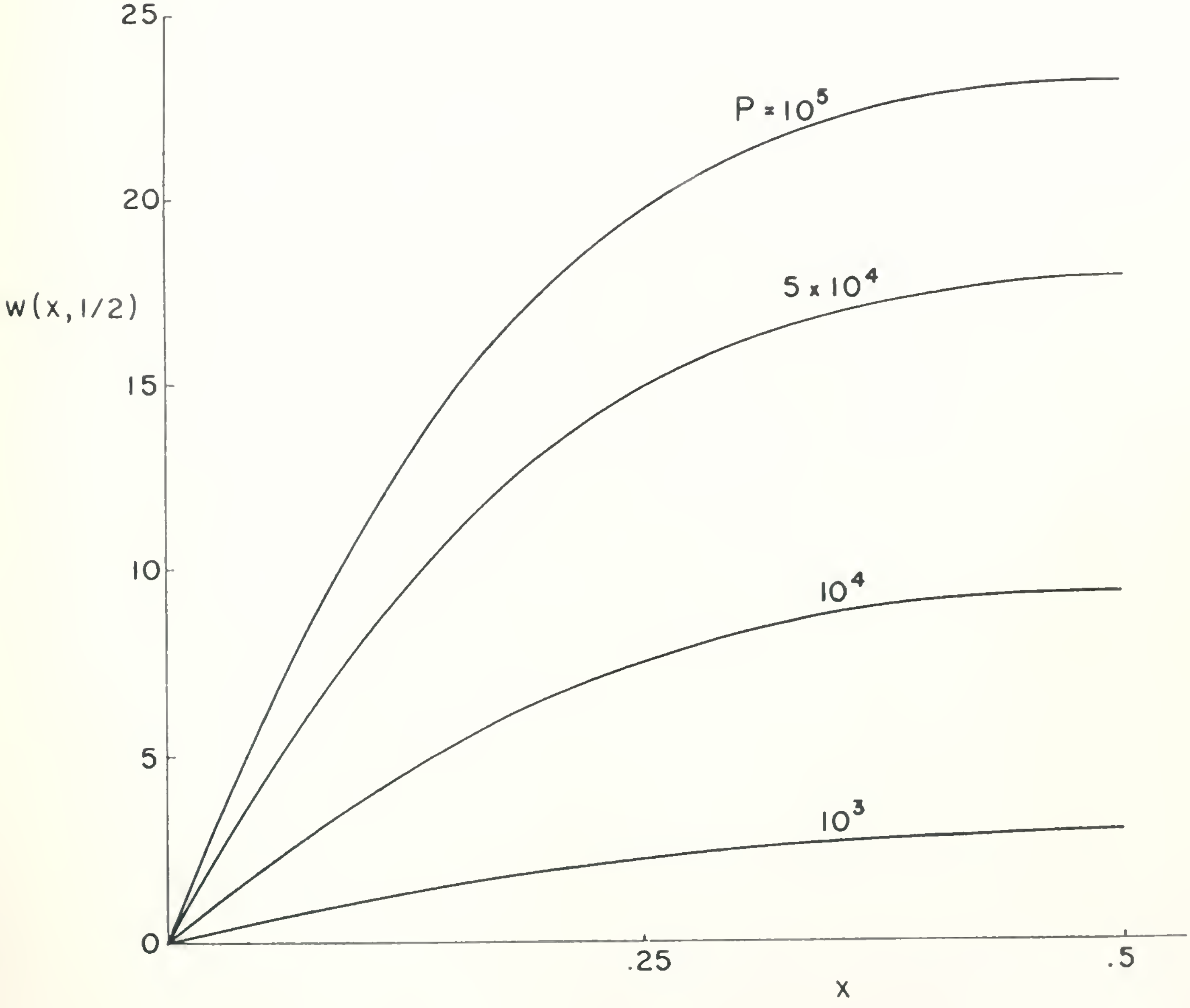

Figure 2a

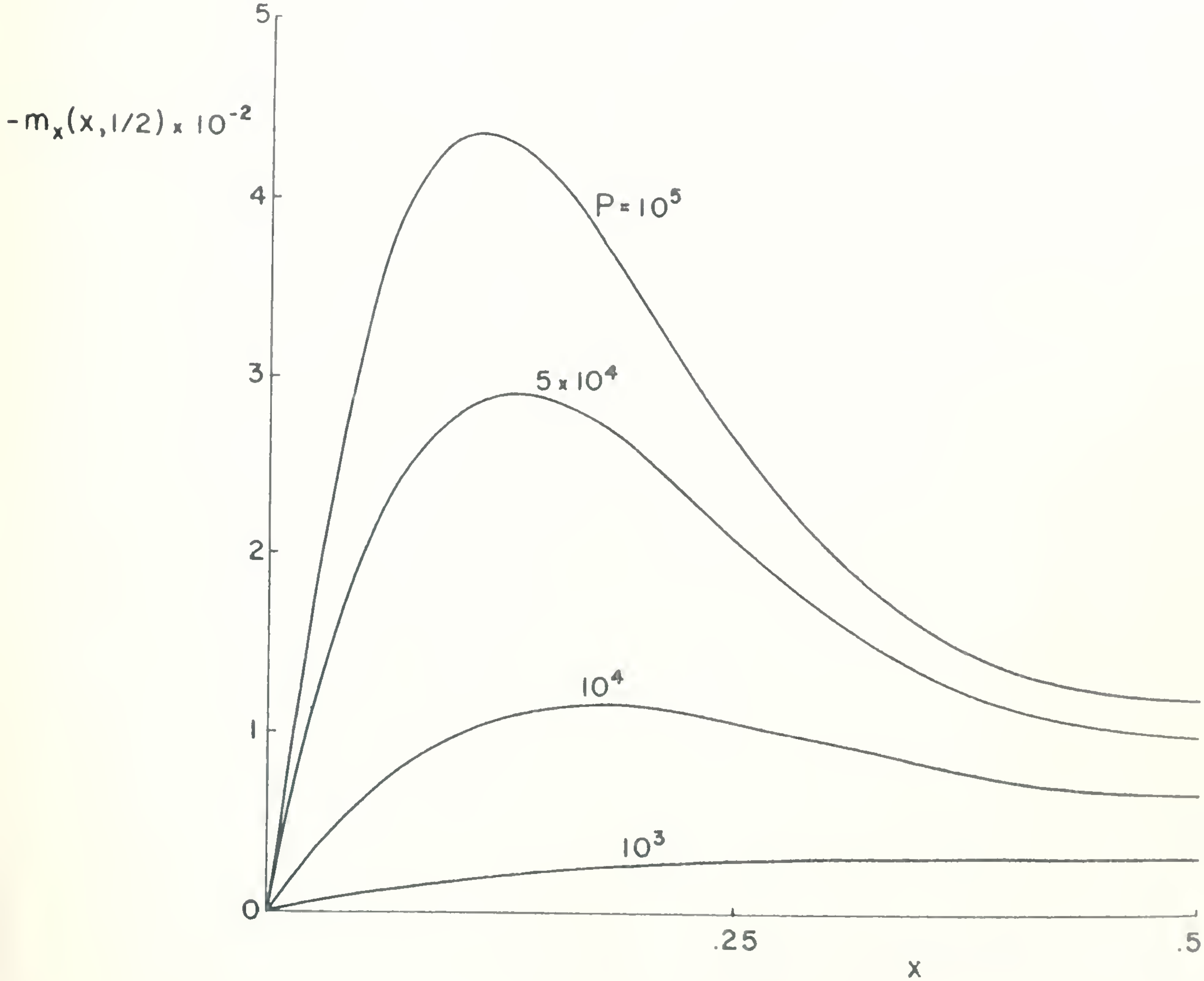

Figure 2b

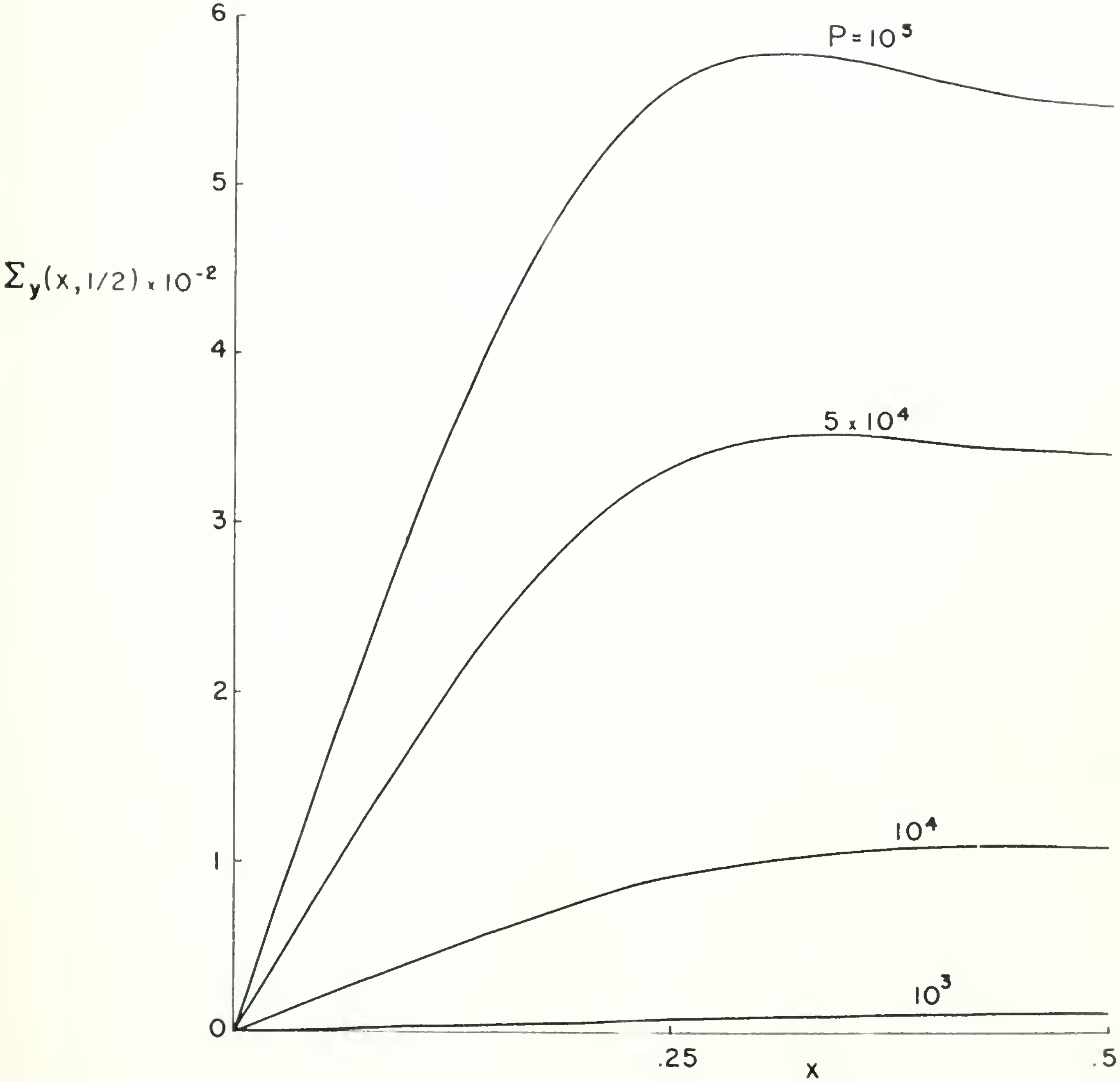

Figure 2c

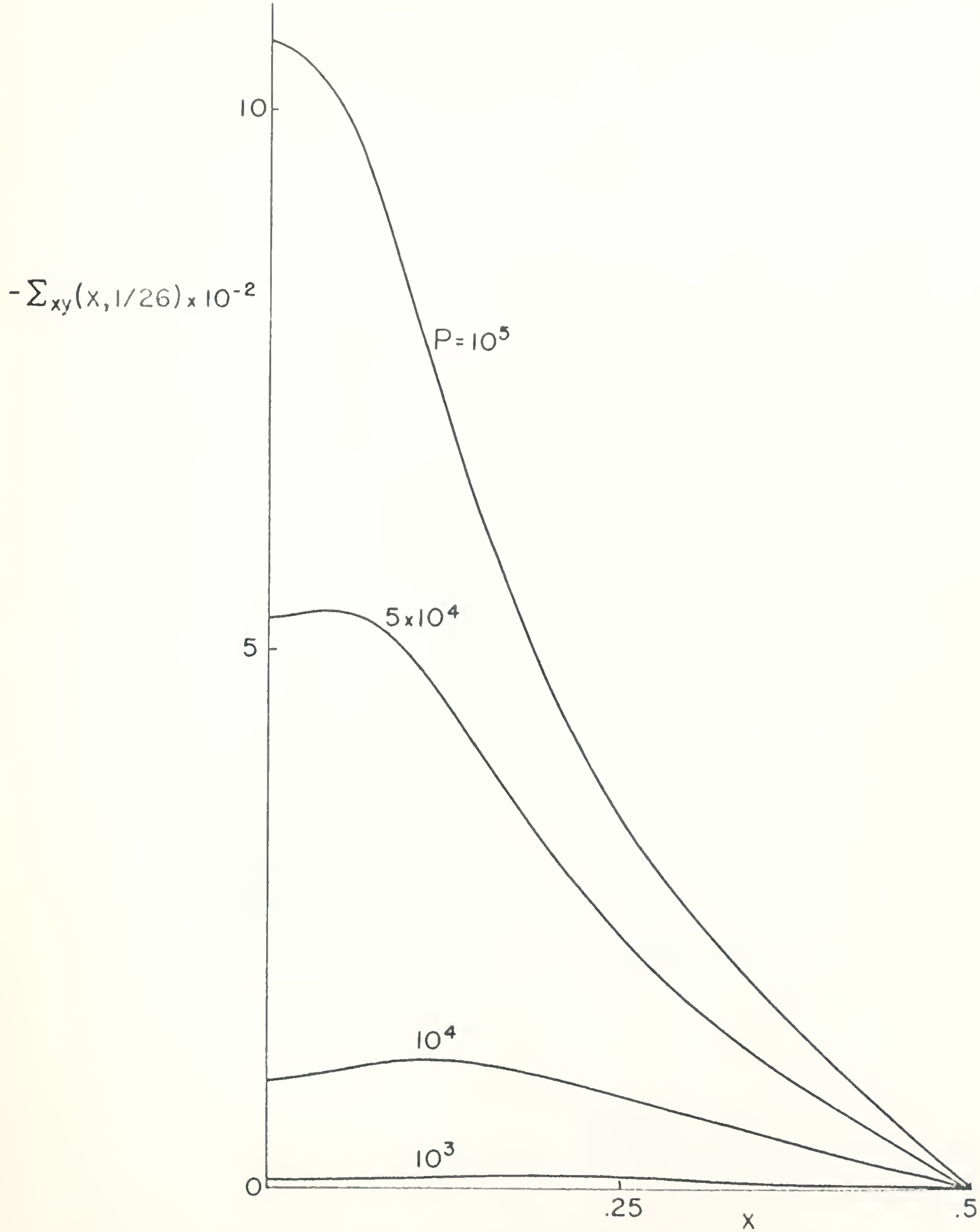

Figure 3a

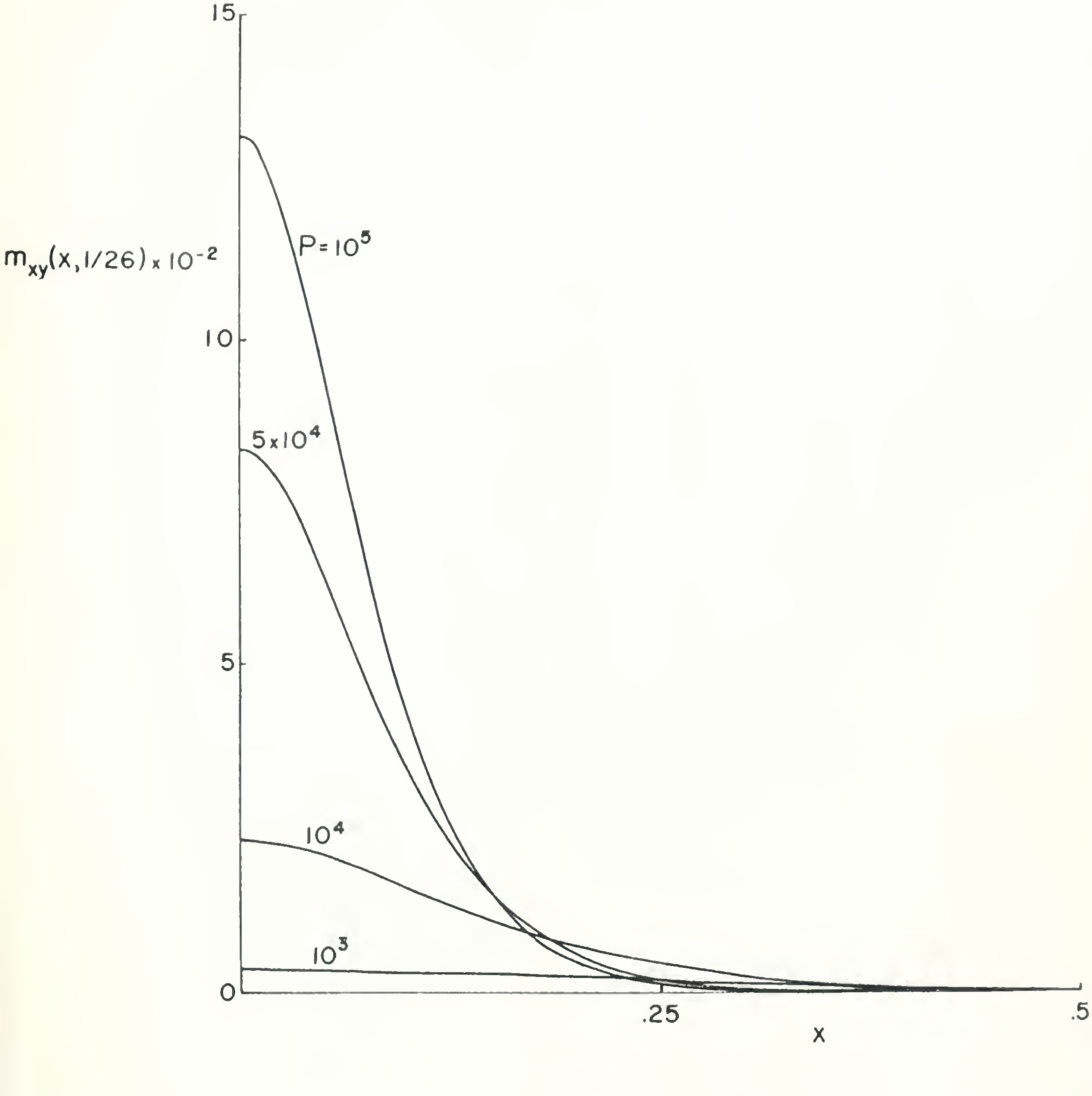

Figure 3b

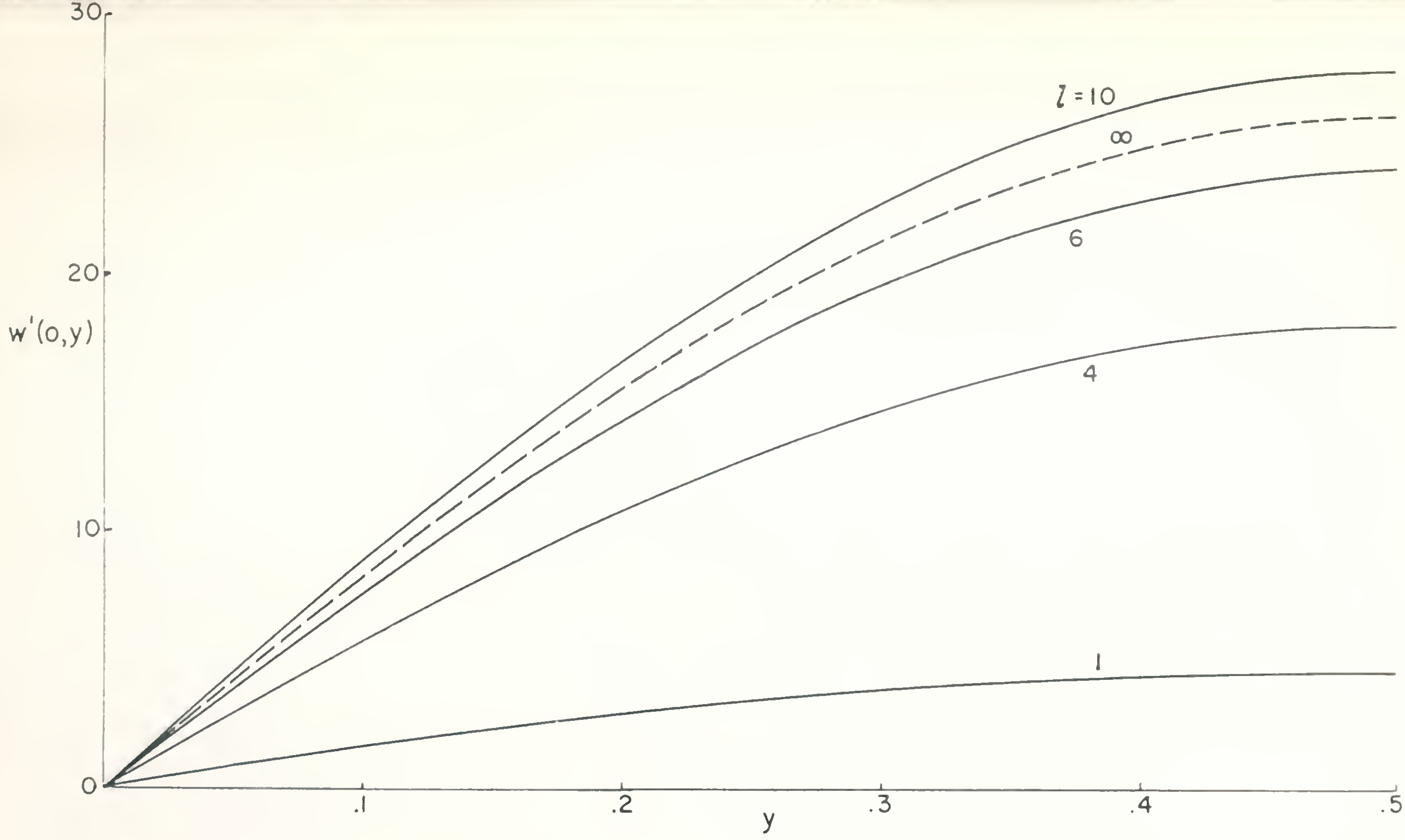

Figure 4a

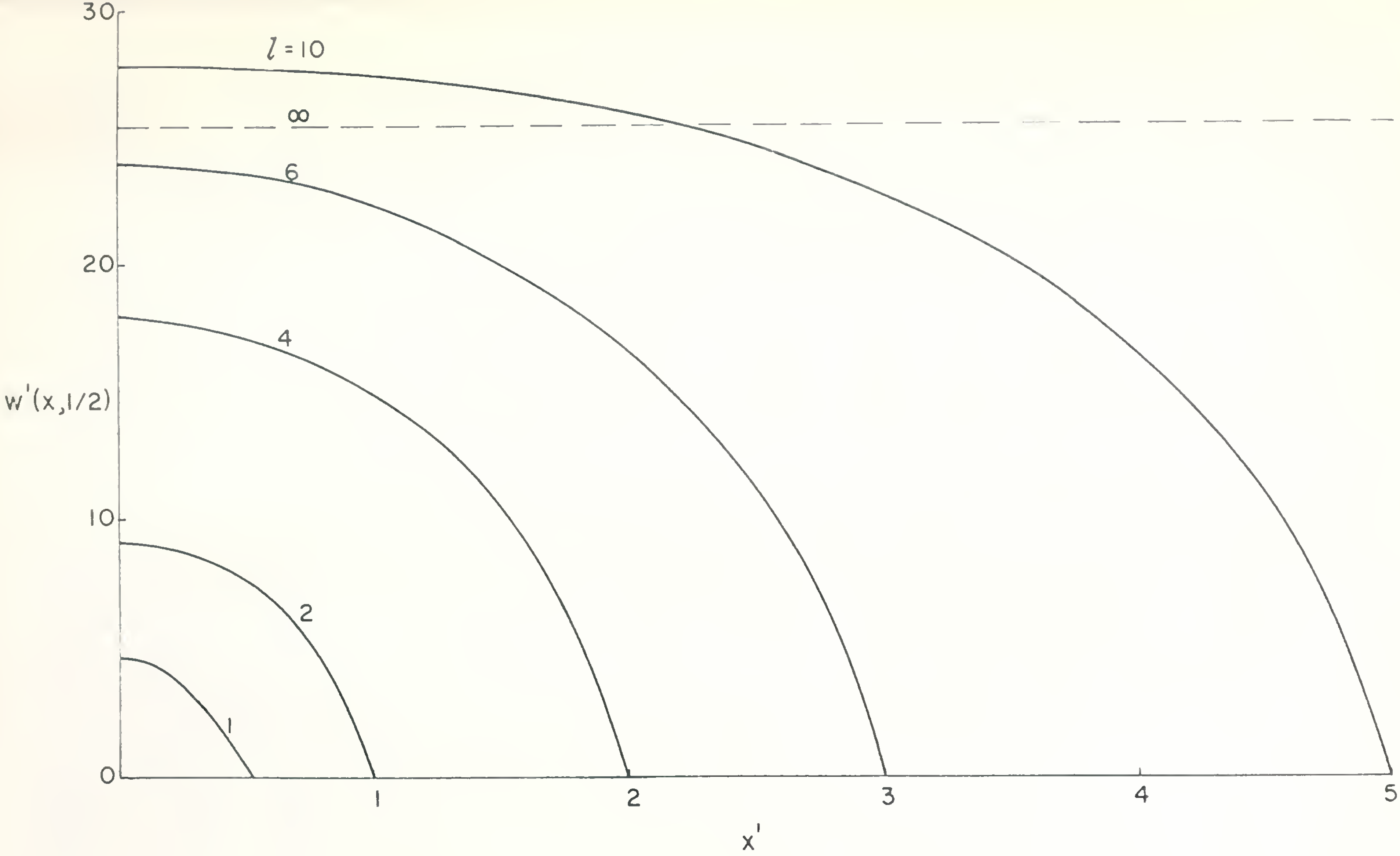

Figure 4b

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